Fundamentals of Accelerators

Lecture - Day 5

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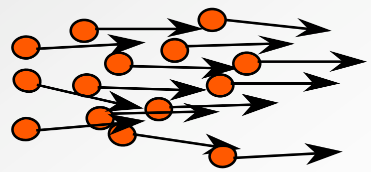
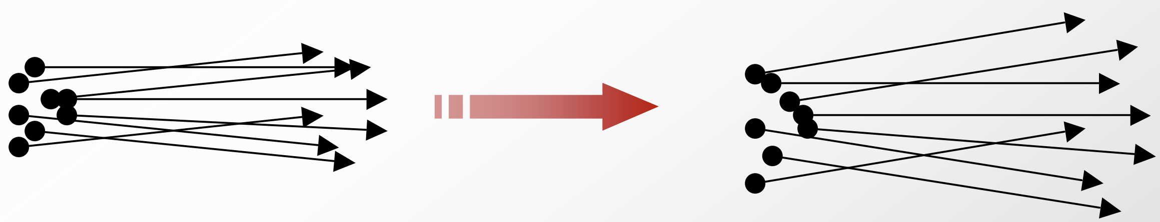
Thermal characteristics of beams

❖ Beams particles have random (thermal) ⊥ motion



$$\vartheta_x = \left\langle \frac{p_x^x}{p_z^2} \right\rangle^{1/2} > 0$$

❖ Beams must be confined against thermal expansion during transport



Brightness of a beam source

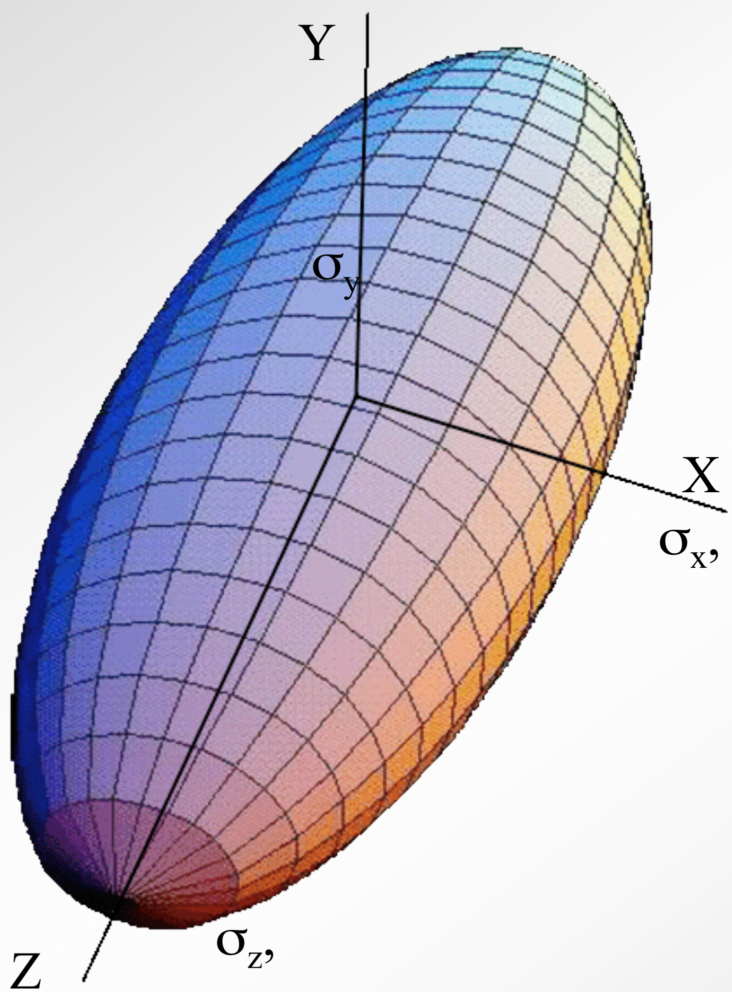
- A figure of merit for the performance of a beam source is the brightness

$$\mathcal{B} = \frac{\text{Beam current}}{\text{Beam area} \circ \text{Beam Divergence}} = \frac{\text{Emissivity (J)}}{\sqrt{\text{Temperature/mass}}}$$

$$= \frac{J_e}{\left(\sqrt{\dfrac{kT}{\gamma m_o c^2}}\right)^2} = \frac{J_e \gamma}{\left(kT/m_o c^2\right)}$$

Typically the normalized brightness is quoted for $\gamma = 1$

Bunch dimensions



For uniform charge distributions

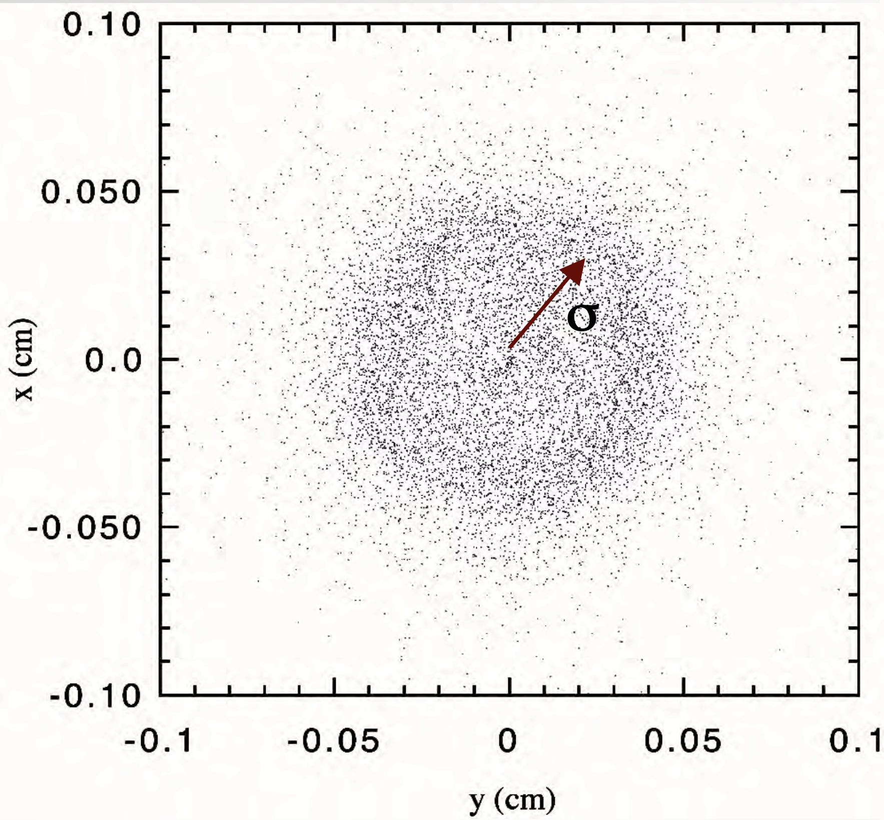
We may use "hard edge values

For gaussian charge distributions

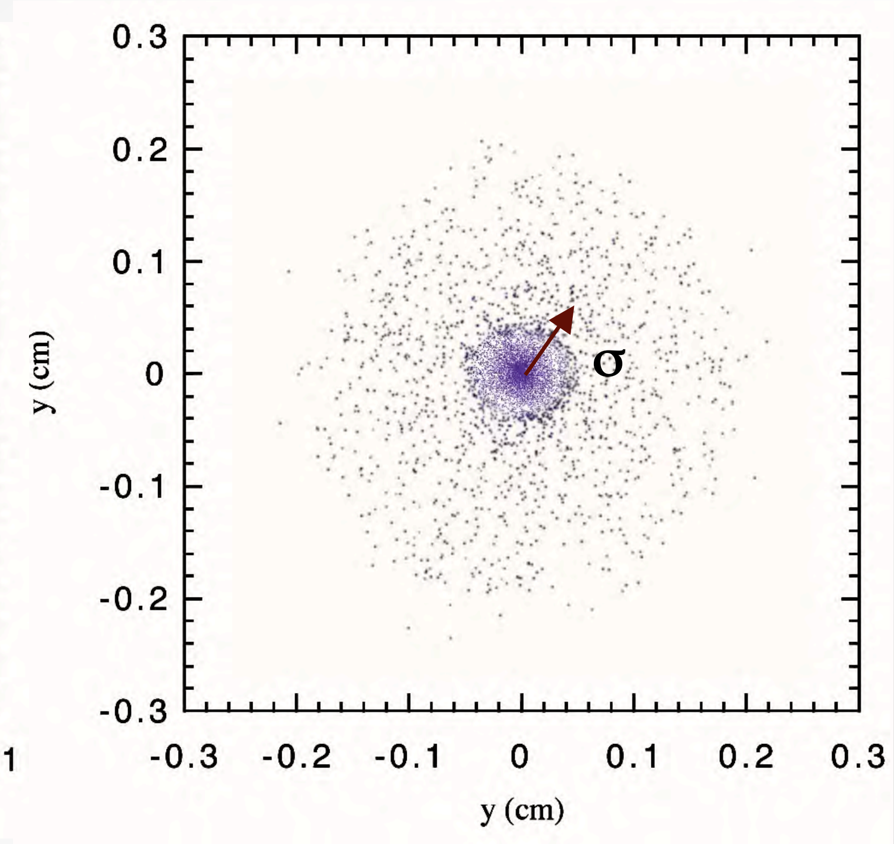
Use rms values σ_x, σ_y, σ_z

We will discuss measurements of bunch size and charge distribution later

But rms values can be misleading



Gaussian beam



Beam with halo

We need to measure the particle distribution

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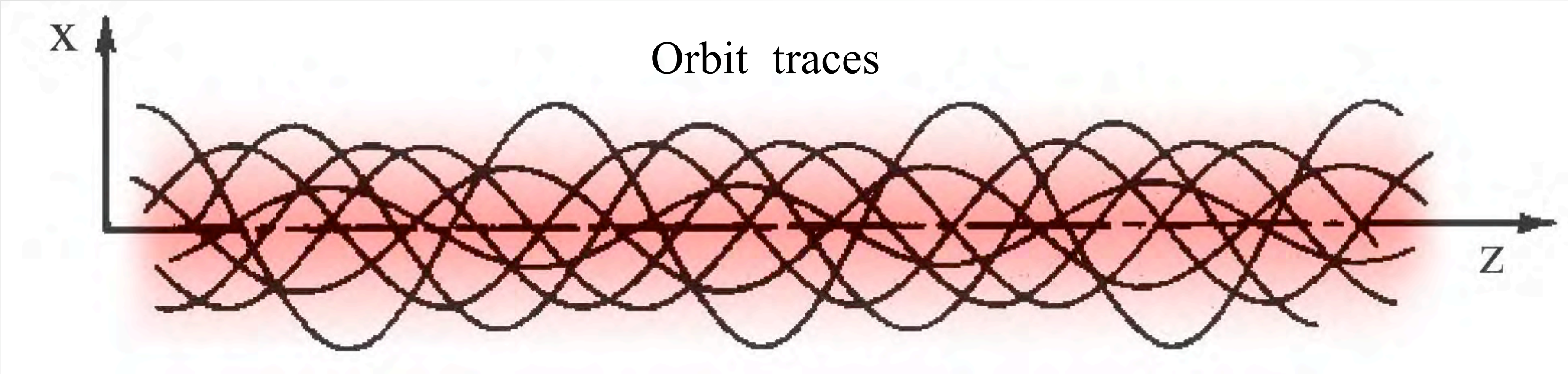
What is this thing called beam quality?
or
How can one describe the dynamics of a bunch of particles?

Coordinate space

Each of N_b particles is tracked in ordinary 3-D space

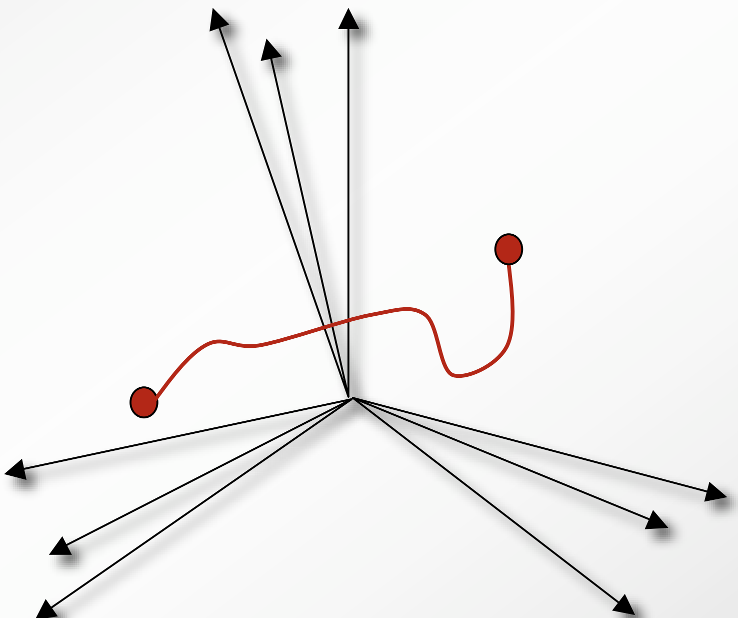


Not too helpful

Configuration space:

$6N_b$-dimensional space for N_b particles; coordinates (x_i, p_i), $i = 1,\ldots, N_b$

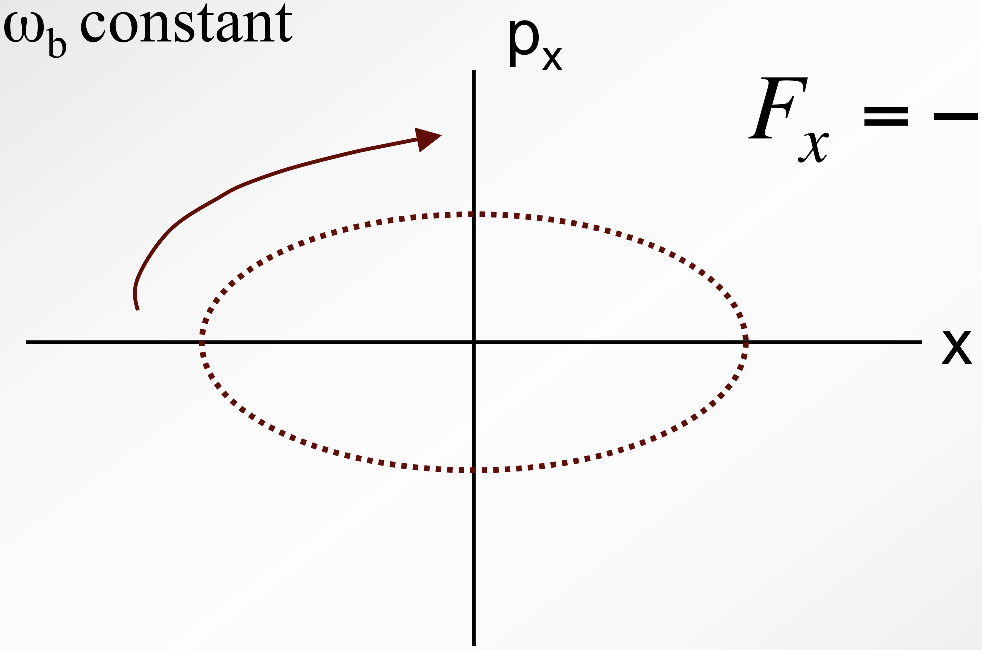
The bunch is represented by a single point that moves in time



Useful for Hamiltonian dynamics

Configuration space example:
One particle in an harmonic potential

ω_b constant



$$F_x = -kx = m\ddot{x}$$

But for many problems this description carries much more information than needed :

We don't care about each of 10^{10} individual particles

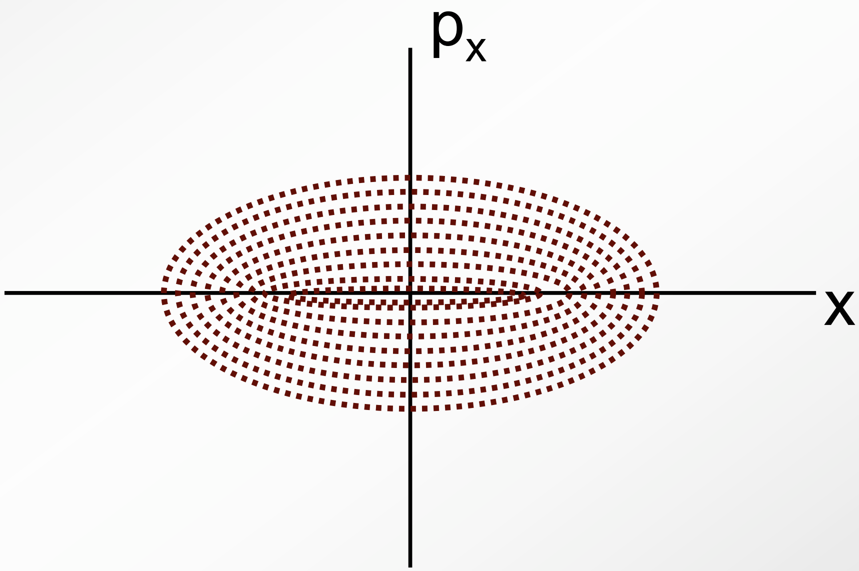
But seeing both the x & p_x looks useful

Option 3: Phase space (gas space in statistical mechanics)

6-dimensional space for N_b particles

The i^{th} particle has coordinates (x_i, p_i), i = x, y, z

The bunch is represented by N_b points that move in time



In most cases, the three planes are to very good approximation decoupled

==> One can study the particle evolution independently in each planes:

Particles Systems & Ensembles

- The set of possible states for a system of N particles is referred as an *ensemble* in statistical mechanics.
- In the statistical approach, particles lose their individuality.
- Properties of the whole system are fully represented by particle density functions f_{6D} and f_{2D} :

$$f_{6D}\left(x, p_x, y, p_y, z, p_z\right)\, dx\, dp_x\, dy\, dp_y\, dz\, dp_z \qquad f_{2D}\left(x_i, p_i\right) dx_i\, dp_i \quad i = 1,2,3$$

where

$$\int f_{6D}\, dx\, dp_x\, dy\, dp_y\, dz\, dp_z = N$$

From: Sannibale USPAS lectures

Longitudinal phase space

- In most accelerators the phase space planes are only weakly coupled.
  - Treat the longitudinal plane independently from the transverse one
  - Effects of weak coupling can be treated as a perturbation of the uncoupled solution
- In the longitudinal plane, electric fields accelerate the particles
  - Use *energy* as longitudinal variable together with its canonical conjugate *time*
- Frequently, we use *relative energy variation* δ & *relative time* τ with respect to a reference particle

$$\delta = \frac{E - E_0}{E_0} \qquad \tau = t - t_0$$

- According to Liouville, in the presence of Hamiltonian forces, the area occupied by the beam in the longitudinal phase space is conserved

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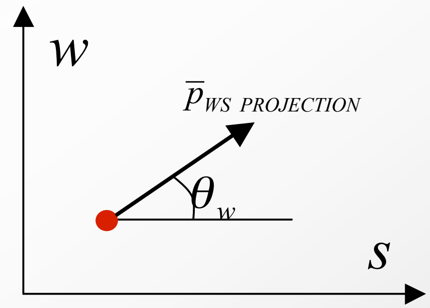
Transverse phase space

- For transverse planes $\{x, p_x\}$ and $\{y, p_y\}$, use a modified phase space where the momentum components are replaced by:

$$p_{xi} \rightarrow x' = \frac{dx}{ds} \qquad\qquad p_{yi} \rightarrow y' = \frac{dy}{ds}$$

where s is the direction of motion



- We can relate the old and new variables (for $Bz \neq 0$) by

$$p_i = \gamma m_0 \frac{dx_i}{dt} = \gamma m_0 v_s \frac{dx_i}{ds} = \gamma \beta m_0 c\, x_i' \qquad \text{i = x,y}$$

$$where \quad \beta = \frac{v_s}{c} \quad and \quad \gamma = \left(1-\beta^2\right)^{-1/2}$$

Note: x_i and p_i are canonical conjugate variables while x and x_i' are not, unless there is no acceleration (γ and β constant)

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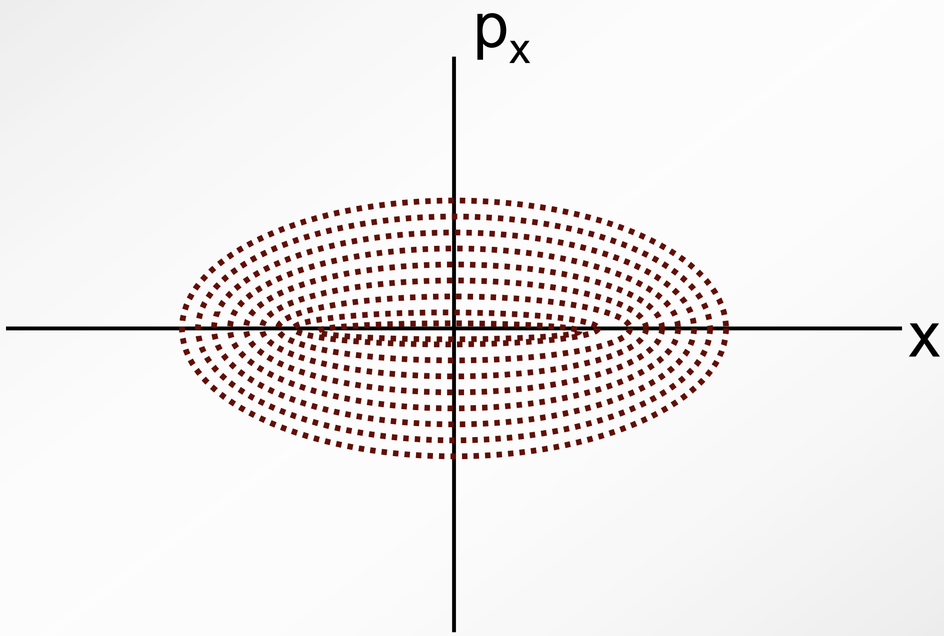
Consider an ensemble of harmonic oscillators in phase space

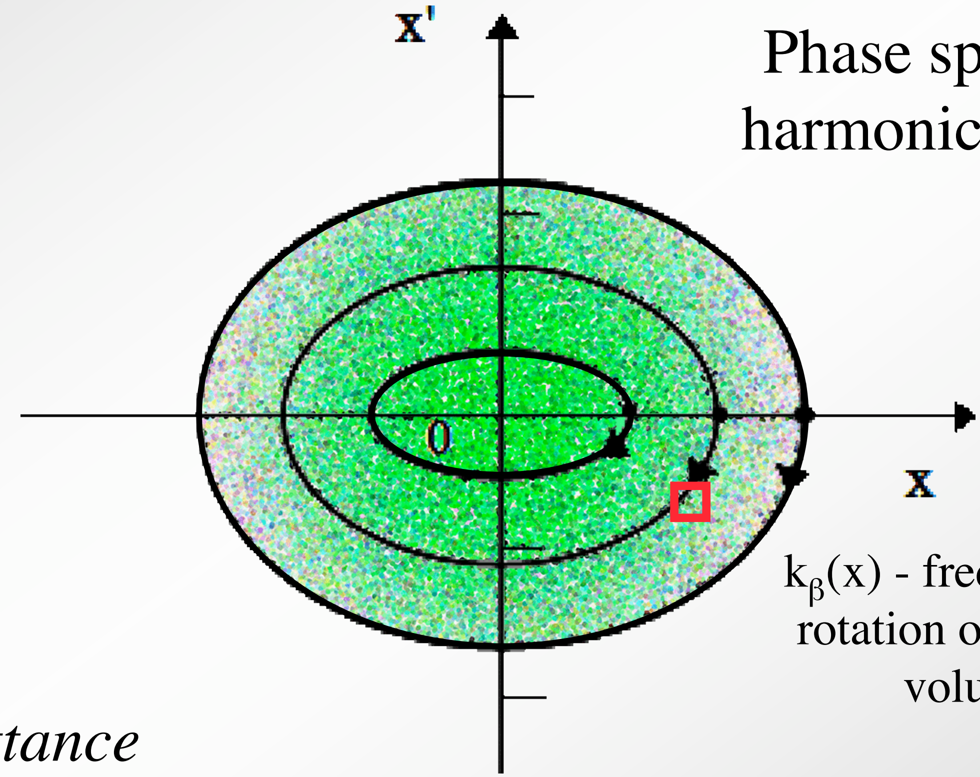


Particles stay on their energy contour.

Again the phase area of the ensemble is conserved

Emittance describes area in phase space of the ensemble of beam particles

Emittance - Phase space volume of beam



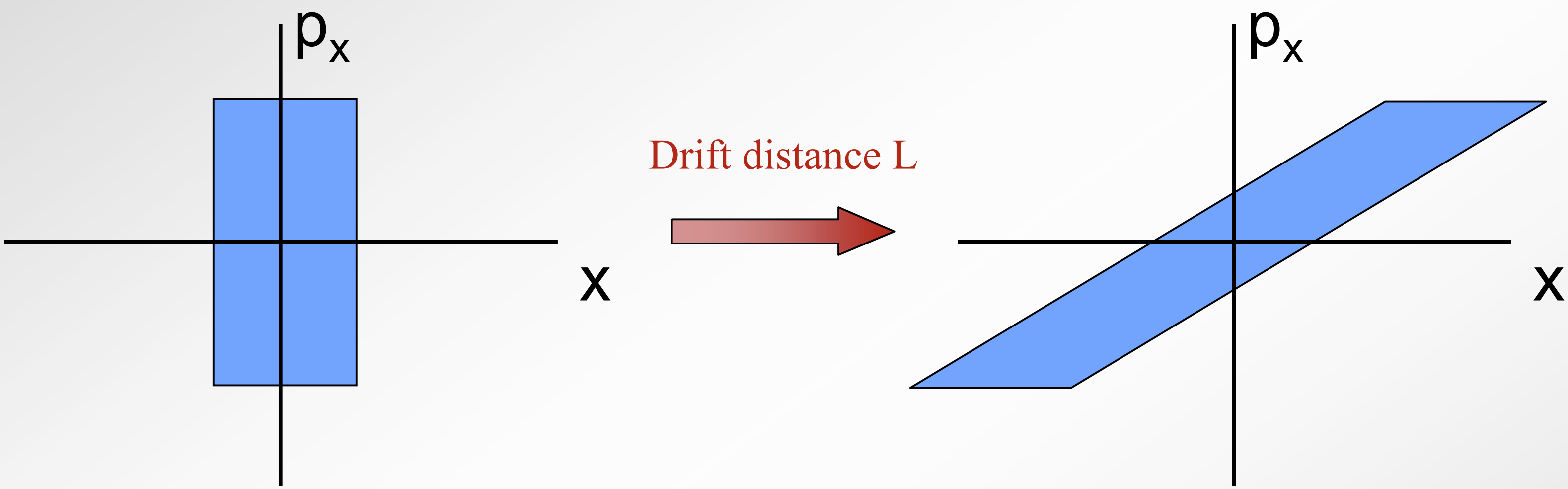
Phase space of an harmonic oscillator

$k_\beta(x)$ - frequency of rotation of a phase volume

RMS emittance

$$\varepsilon^2 \equiv R^2 (V^2 - (R')^2)/c^2$$

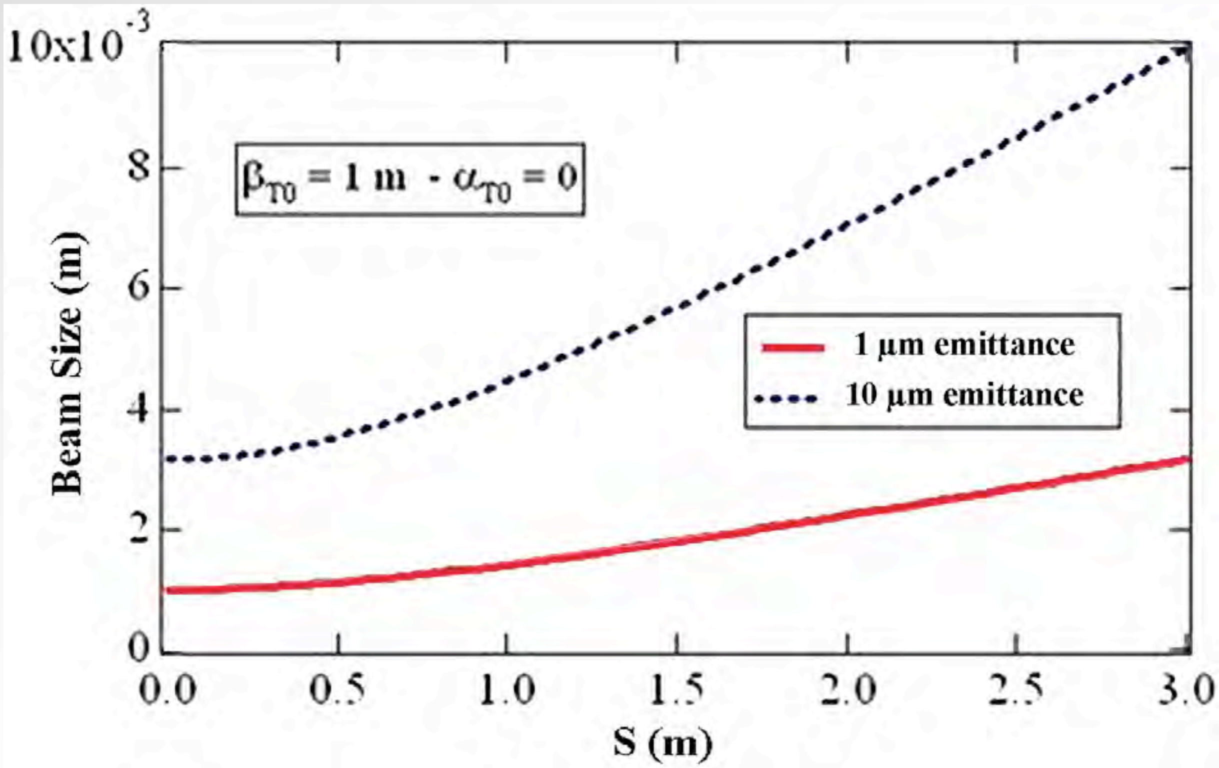
Force-free expansion of a beam



Notice: The phase space area is conserved

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \Rightarrow \begin{matrix} x = x_0 + Lx_0' \\ x' = x_0' \end{matrix}$$

A numerical example:
Free expansion of a due due to emittance

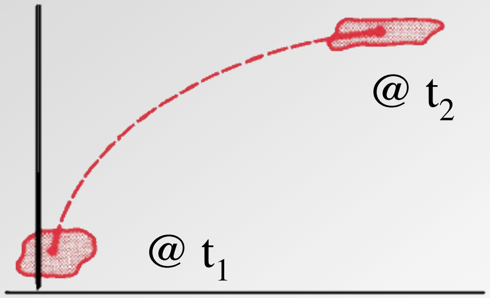


$$R^2 = R_o^2 + V_o^2 L^2 = R_o^2 + \frac{\varepsilon^2}{R_o^2} L^2$$

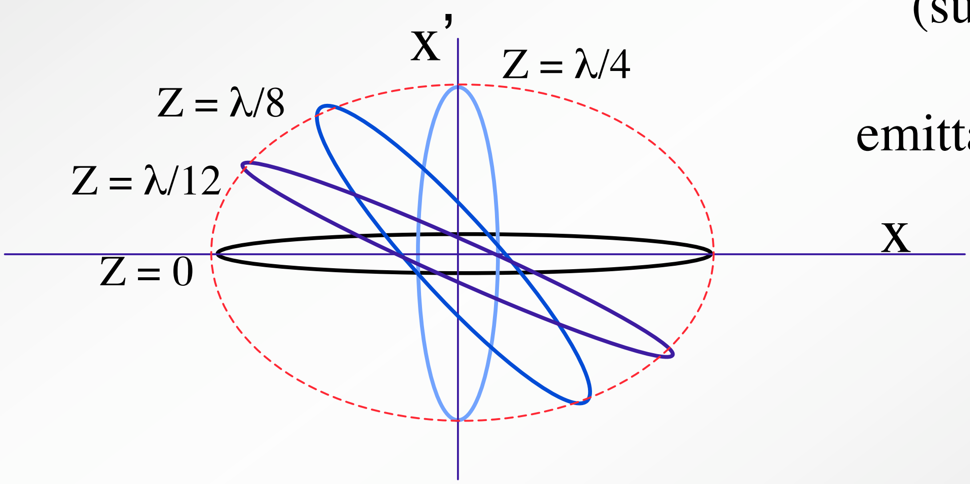
This emittance is the phase space area occupied by the system of particles, divided by π

The rms emittance is a measure of the mean non-directed (thermal) energy of the beam

Why is emittance an important concept



1) Liouville: Under conservative forces phase space evolves like an incompressible fluid ==>

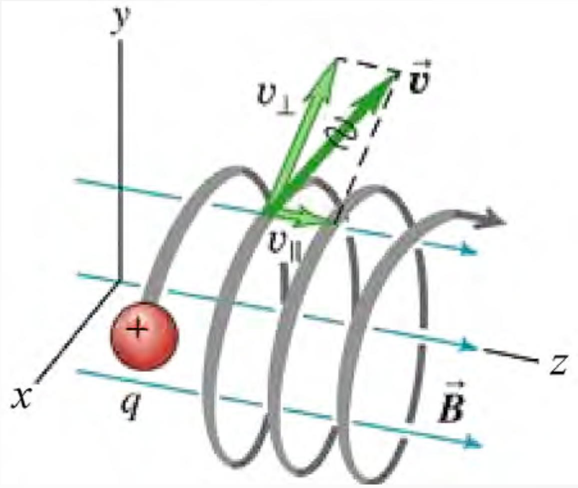


2) Under linear forces macroscopic (such as focusing magnets) & γ =constant emittance is an invariant of motion

3) Under acceleration $\gamma\varepsilon = \varepsilon_n$ is an adiabatic invariant

Emittance conservation with B_z

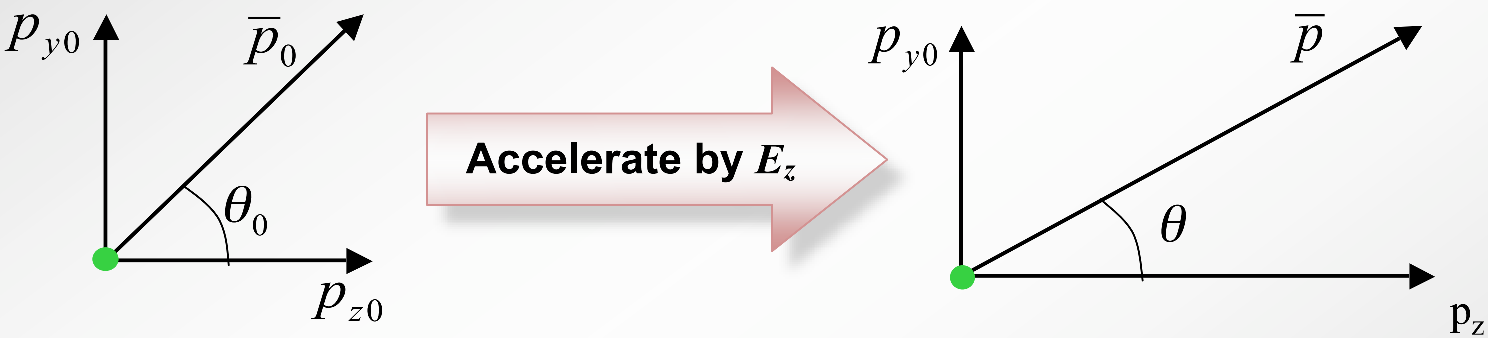
- An axial B_z field, (e.g.,solenoidal lenses) couples transverse planes
  - The 2-D Phase space area occupied by the system in each transverse plane is no longer conserved



- Liouville's theorem still applies to the 4D transverse phase space
  - the 4-D hypervolume is an invariant of the motion
- In a frame rotating around the *z* axis by the *Larmor frequency* $\omega_L = qB_z / 2g\ m_0$, the transverse planes decouple
  - The phase space area in each of the planes is conserved again

Emittance during acceleration

- When the beam is accelerated, β & γ change
  - x and x' are no longer canonical
  - Liouville theorem does not apply & emittance is not invariant



$$p_z = \sqrt{\frac{T^2 + 2Tm_0c^2}{T_0^2 + 2T_0m_0c^2}}\, p_{z0}$$

$$T \equiv kinetic\ energy$$

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Then…

$$y_0' = \tan\theta_0 = \frac{p_{y0}}{p_{z0}} = \frac{p_{y0}}{\beta_0 \gamma_0 m_0 c} \qquad y' = \tan\theta = \frac{p_y}{p_z} = \frac{p_{y0}}{\beta \gamma m_0 c} \qquad \frac{y'}{y_0'} = \frac{\beta_0 \gamma_0}{\beta \gamma}$$

In this case $\frac{\varepsilon_y}{\varepsilon_{y0}} = \frac{y'}{y_0'}$ ==> $\boxed{\beta \gamma \varepsilon_y = \beta_0 \gamma_0 \varepsilon_{y0}}$

- Therefore, the quantity $\beta \gamma \varepsilon$ is invariant during acceleration.
- Define a conserved *normalized emittance*

$$\boxed{\varepsilon_{n\,i} = \beta \gamma \varepsilon_i \qquad i = x, y}$$

Acceleration couples the longitudinal plane with the transverse planes

The 6D emittance is still conserved but the transverse ones are not

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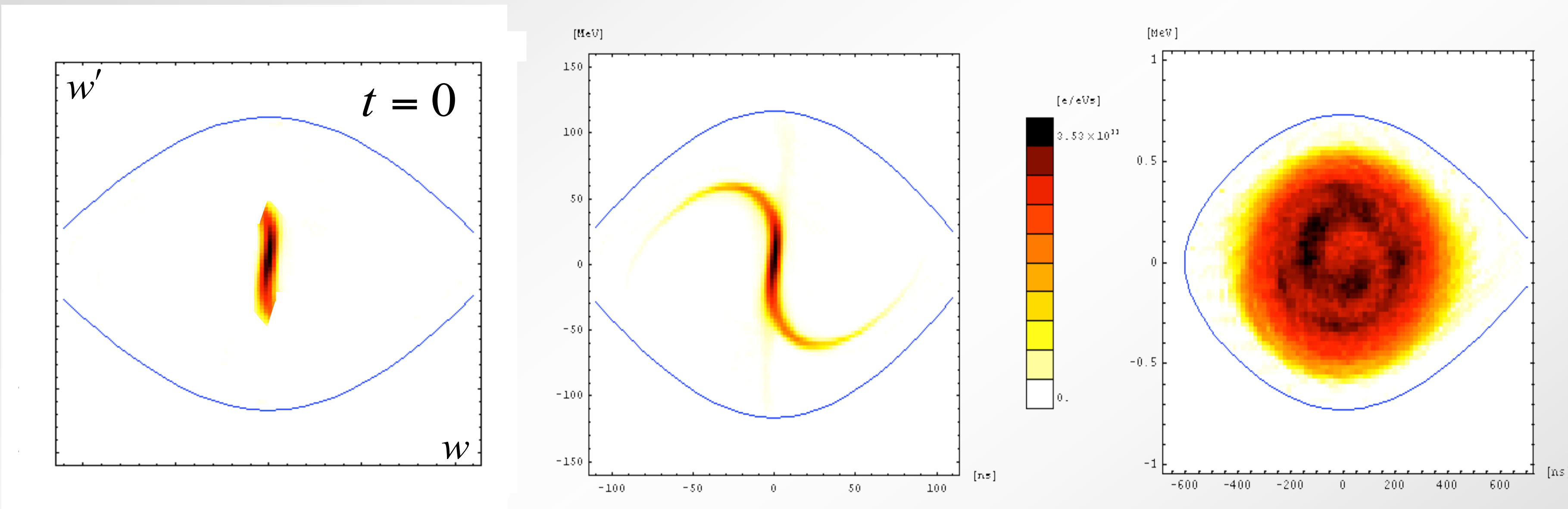
Example 2: Filamentation of longitudinal phase space

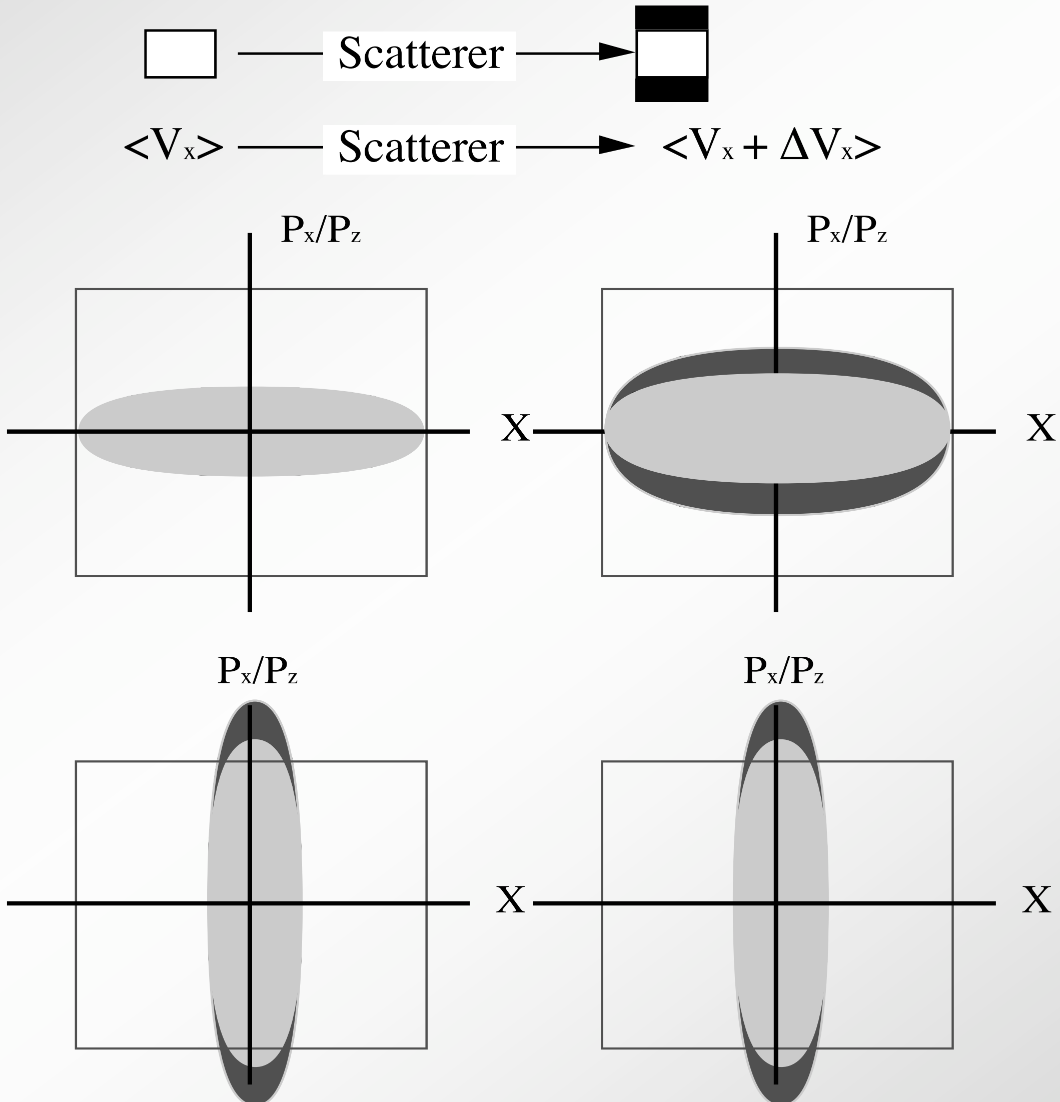


Data from CERN PS

The emittance according to Liouville is still conserved

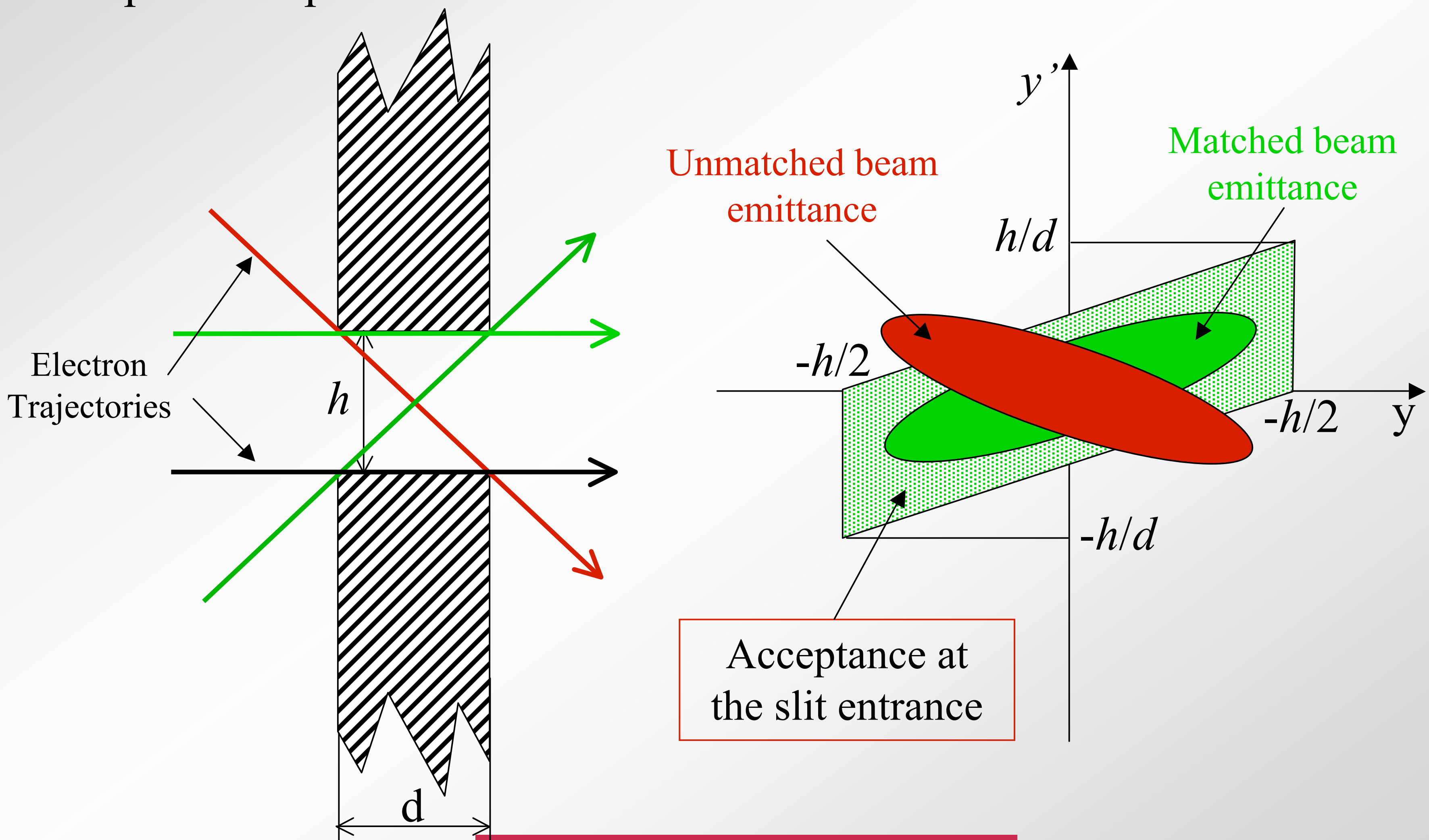
Macroscopic (rms) emittance is not conserved

Non-conservative forces (scattering) increases emittance



The Concept of Acceptance

Example: Acceptance of a slit



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Measuring the emittance of the beam

$$\varepsilon^2 = R^2 (V^2 - (R')^2) / c^2$$

- RMS emittance
  - Determine rms values of velocity & spatial distribution
- Ideally determine distribution functions & compute rms values
- Destructive and non-destructive diagnostics

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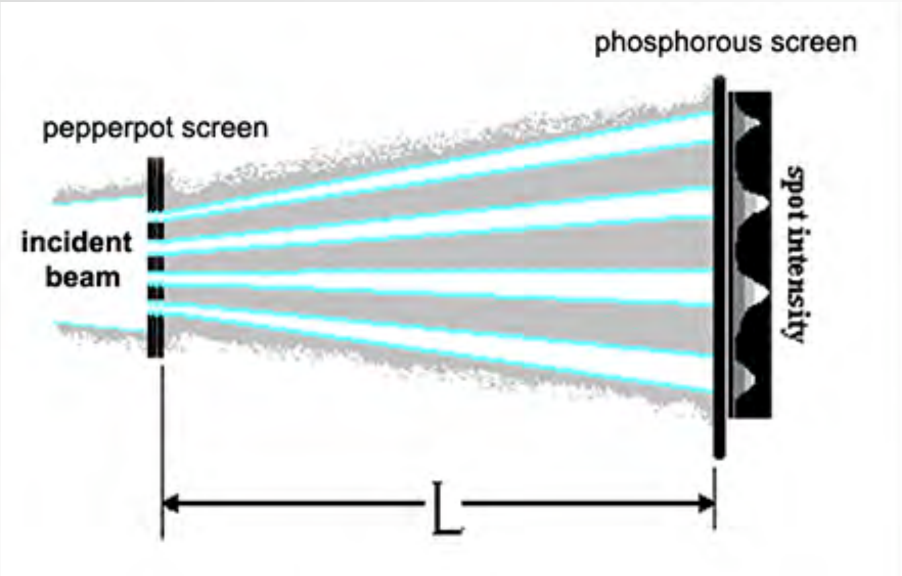
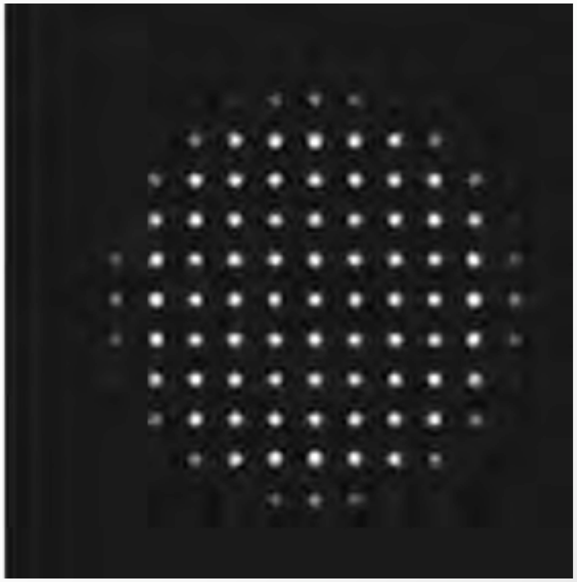
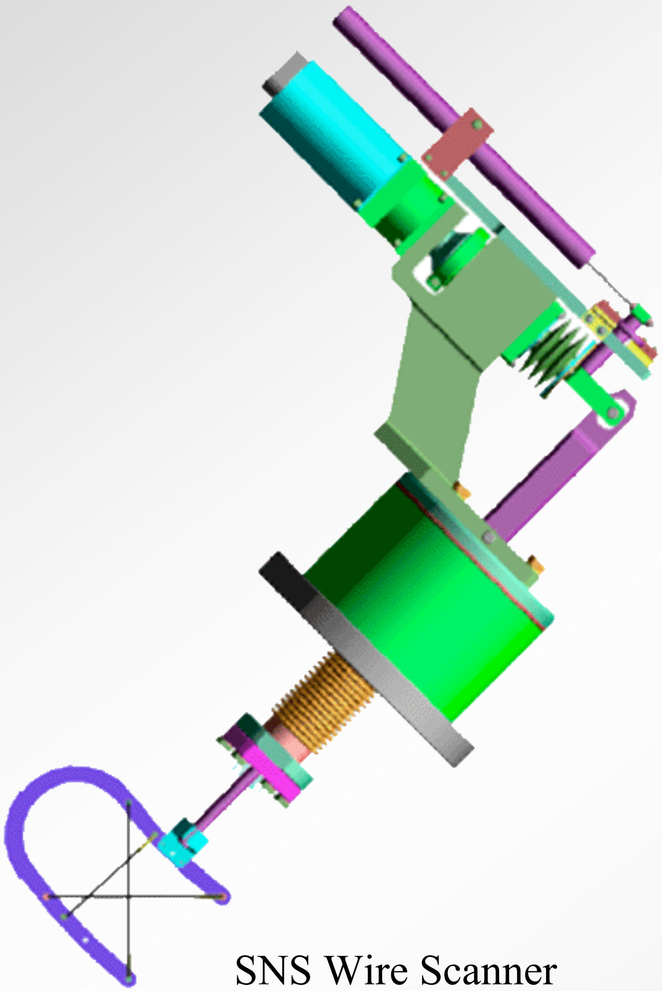
Example of pepperpot diagnostic





- Size of image ==> R
- Spread in overall image ==> R′
- Spread in beamlets ==> V
- Intensity of beamlets ==> current density

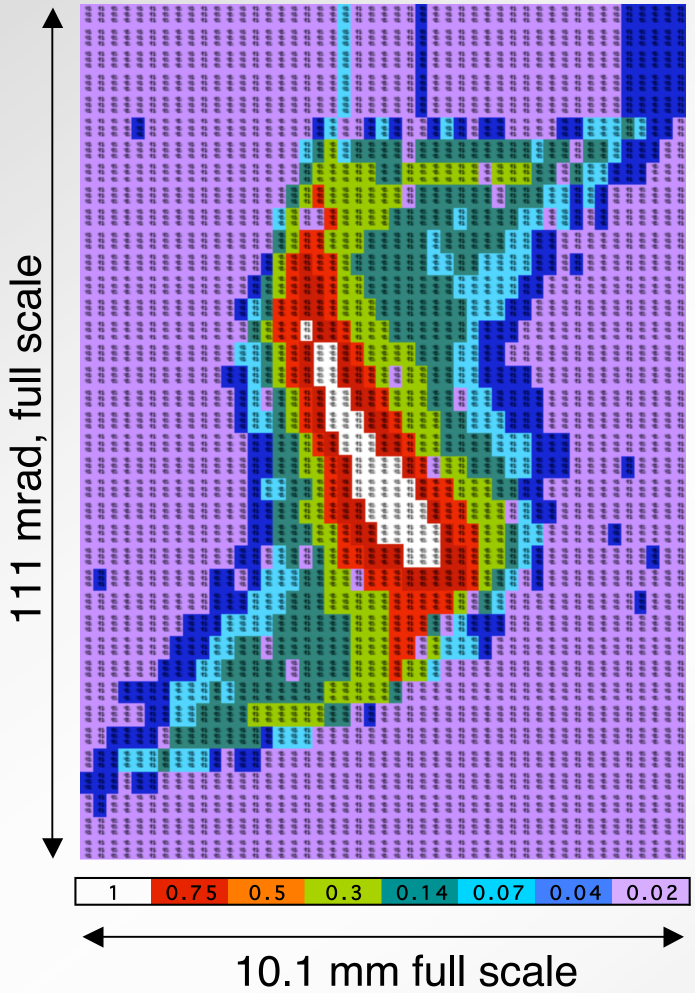
Wire scanning to measure R and ε



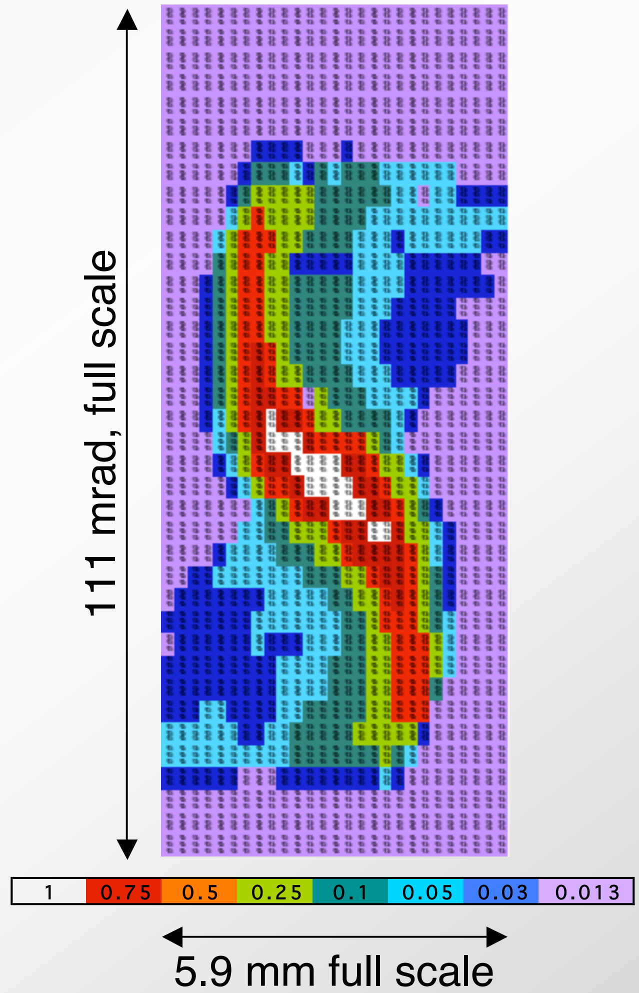
SNS Wire Scanner

- Measure x-ray signal from beam scattering from thin tungsten wires
- Requires at least 3 measurements along the beamline

Measured 33-mA Beam RMS Emittances



Horizontal, 0.22 pi mm mrad



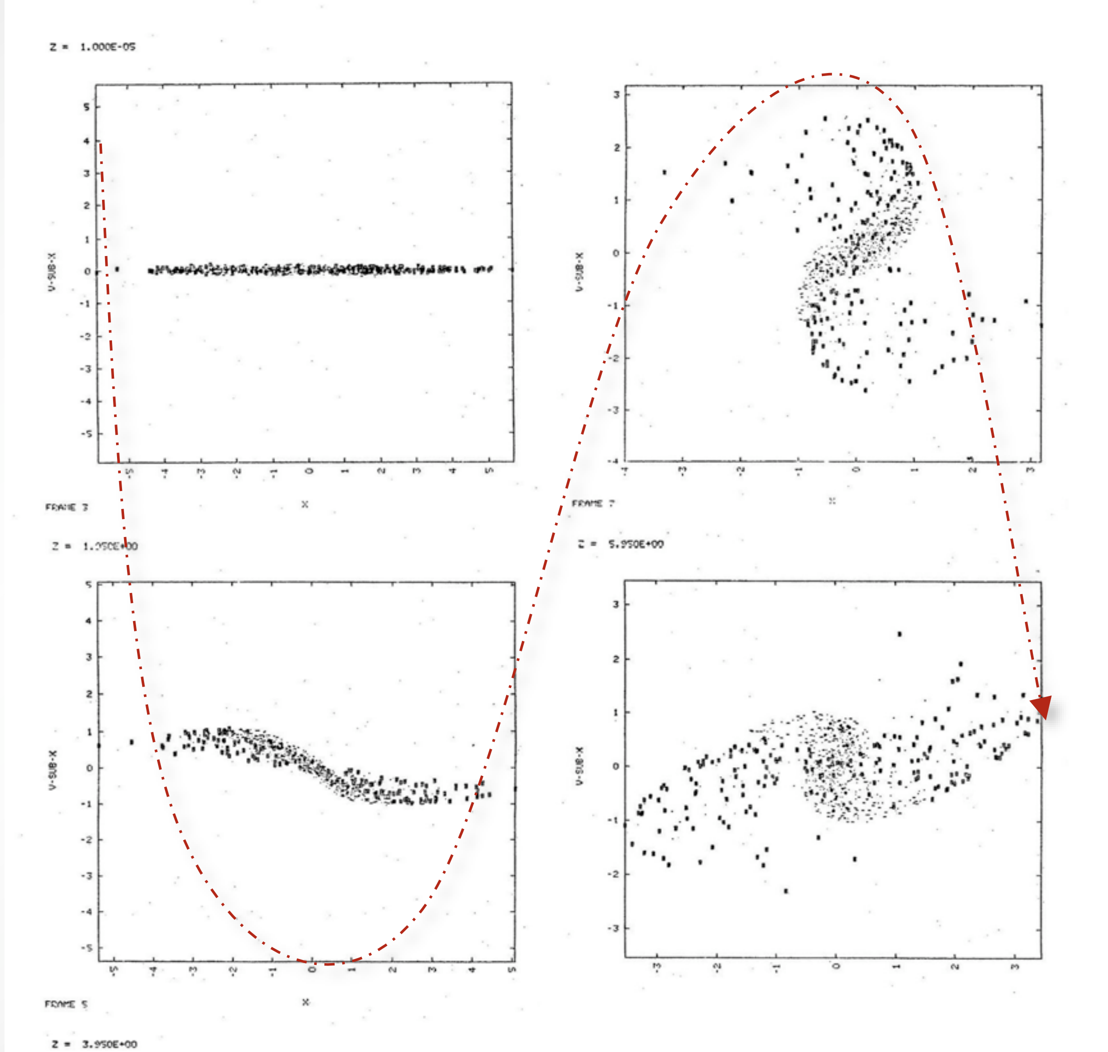
Vertical, 0.15 pi mm mrad

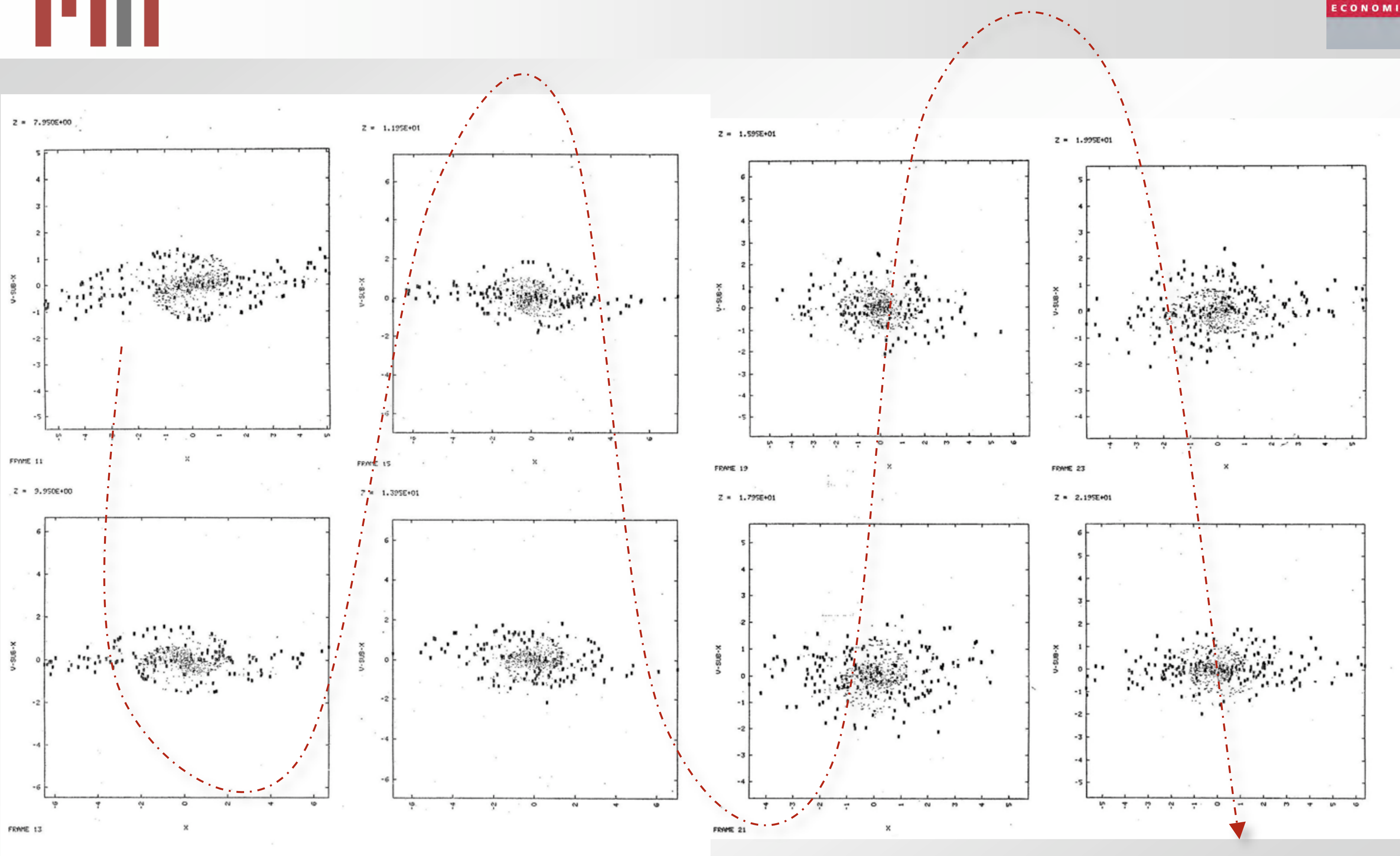
Nonlinear space-charge fields filament phase space via Landau damping

Consider a cold beam with a Gaussian charge distribution entering a dense plasma

At the beam head the plasma shorts out the E_r leaving only the azimuthal B-field

The beam begins to pinch trying to find an equilibrium radius




Z = 7.950E+00
Z = 1.195E+01
Z = 1.595E+01
Z = 1.995E+01
FRAME 11
FRAME 15
FRAME 19
FRAME 23
Z = 9.950E+00
Z = 1.395E+01
Z = 1.795E+01
Z = 2.195E+01
FRAME 13
FRAME 21
V-SUB-X
X

Is there any way to decrease the emittance?

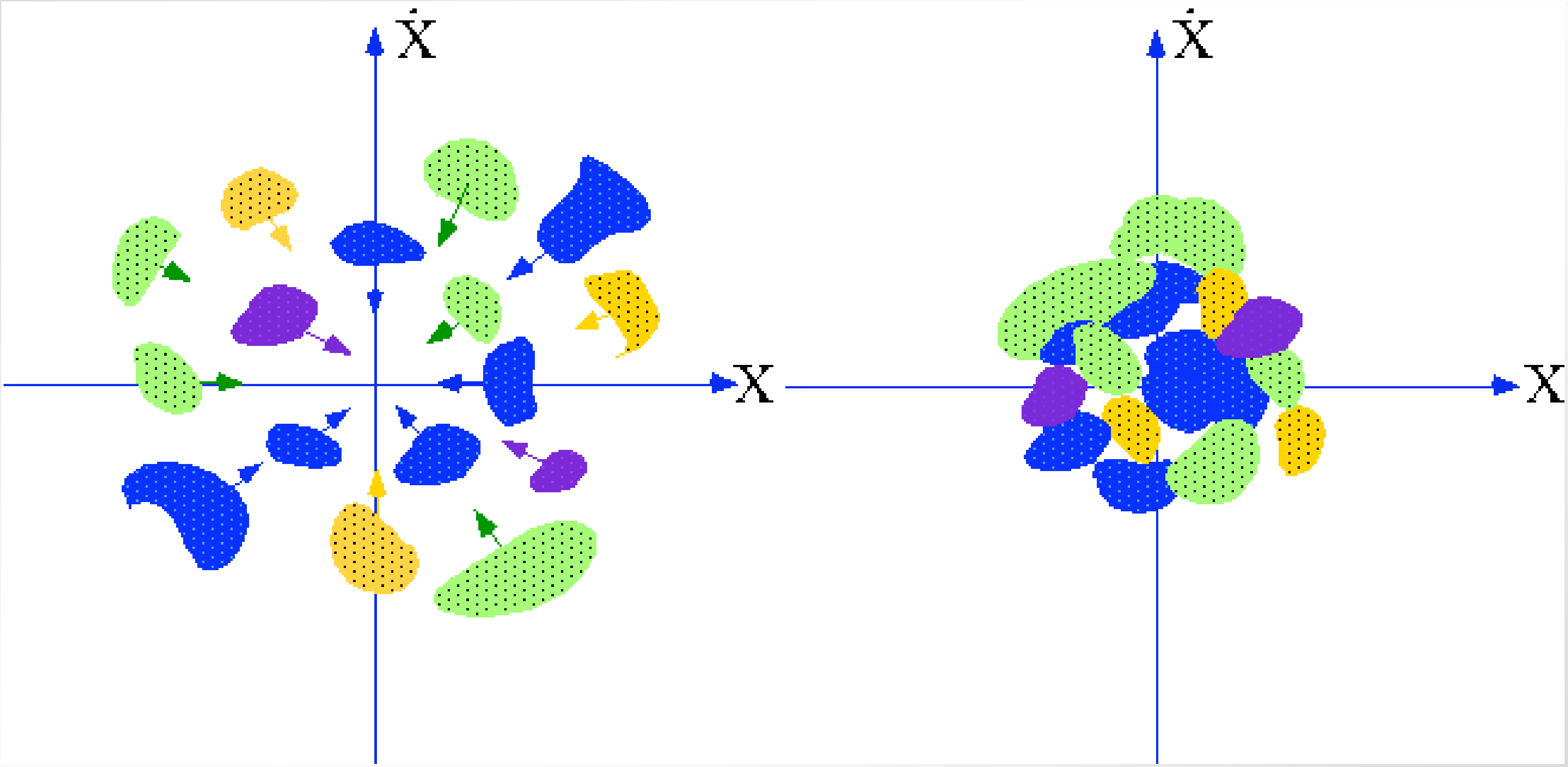
This means taking away mean transverse momentum,

but

keeping mean longitudinal momentum

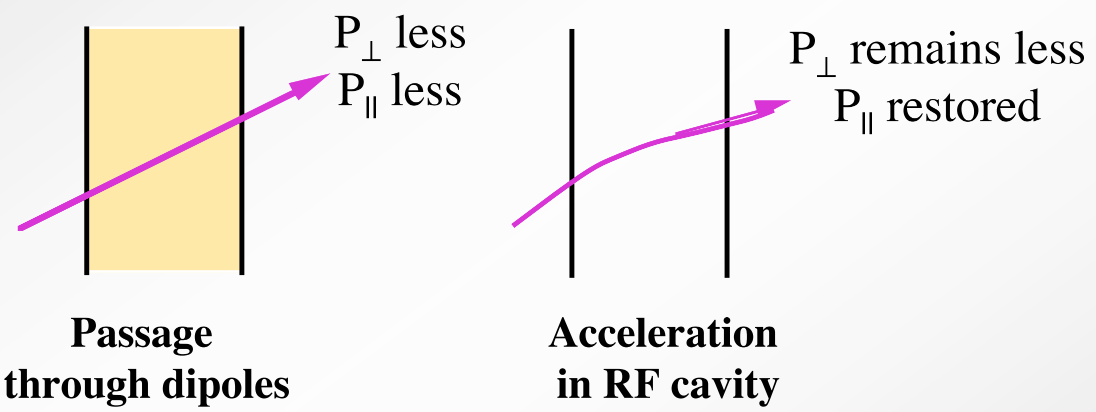
We'll leave the details for later in the course.

Phase-Space Cooling in Any One Dimension



Schematic: radiation & ionization cooling

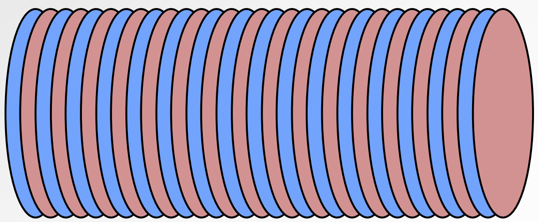
Transverse cooling:

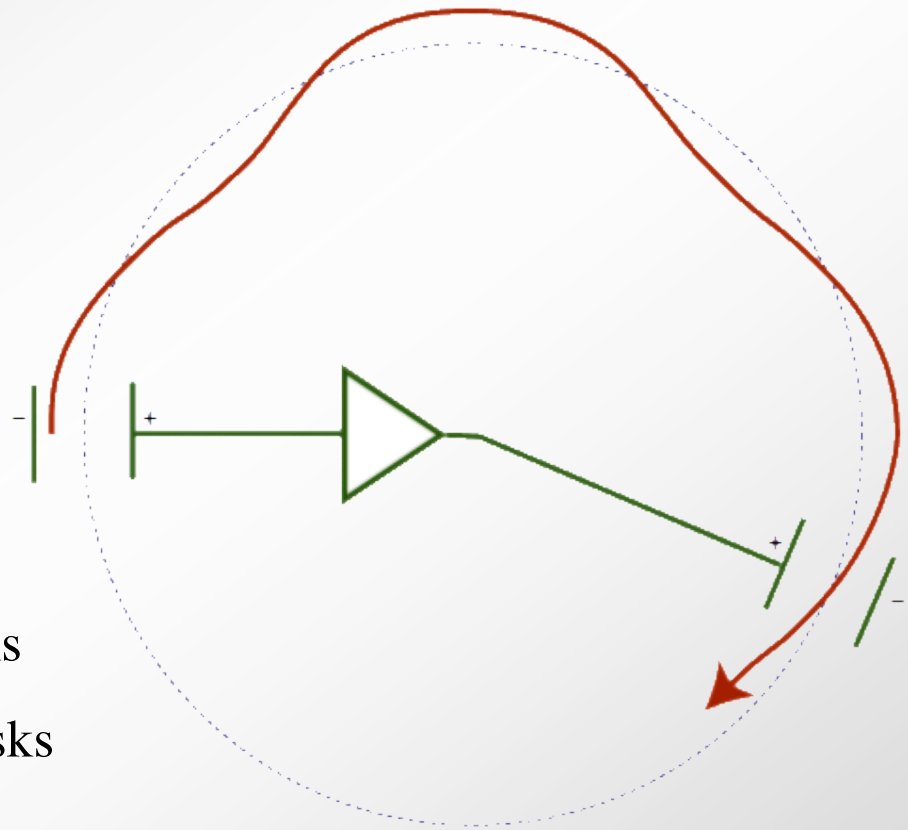


Limited by quantum excitation

Cartoon of transverse stochastic cooling

Van der Meer Nobel prize





Divide (sample) the beam into disks

1) rf pick-up sample centroid of disks

2) Kicker electrode imparts $v_\perp$

to center the disk